ECONOMICS AND MATHEMATICS

by

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Introduction

We can define pure mathematics as the investigation, by conceptual (*a priori*) means, of problems concerning conceptual systems, or members of such, with the aim of finding (inventing or discovering) the patterns satisfied by such objects - a finding justified only by rigorous proof.

We assume that pure mathematics, as classically exemplified by arithmetic, geometry, or analysis, is a formal research field, i.e. one such that all of its objects are constructs, and all of its truth claims must be sustained by purely conceptual means. This conceptualist thesis characterises Bunge's philosophy of mathematics as sketched in his *Treatise* (1974a,b; 1985). This thesis can be traced back to Plato. Indeed, Plato was the first to recognize the ideal (or conceptual) nature of mathematical objects, as well as the purely conceptual character of mathematical procedures (see Wedberg 1955). However, Bunge's conceptualist thesis is different from Plato's doctrine of forms or pure ideas. Far from holding that ideas exist by themselves in a realm of their own, Bunge holds (1979, chapter 2, section 4.3 and 1983a, chapter 1, section 2.1) ideas to be brain processes, and mathematical constructs to be equivalence classes thereof. Bunge's is thus a methodological and not an ontological dualism. In short, when doing mathematics we proceed *as if* we were Platonists: we pretend that mathematical objects, and the formulas about them, exist on their own. This pretense entails Bunge's distinguishing mathematical statements from statements about knowledge, as the latter belong to epistemology.

Mathematics and reality

Considered as a conceptual system, mathematics has no factual content and makes no essential use of empirical procedures.

Throughout his *Treatise* Bunge distinguishes constructs, such as concepts and propositions, from factual items, such as tangible things. Thus it is held that constructs have peculiar - mathematical and semantic - properties that factual items lack (see Bunge 1974a, vol. 1, chapter 1). But at the same time it is maintained that all constructs are created by rational animals and, more precisely, that they may be constructed as equivalence classes of brain processes of a certain type (Bunge 1983, vol. 5, chapter 1). Therefore this construct/fact dichotomy is methodological, not ontological.

Distinguishing constructed from factual items amounts to distinguishing *formal* (conceptual, ideal) *existence* from *factual* (concrete, material) *existence*. For instance, whereas numbers exist formally, protons exist factually. Numbers are members of a collection of constructs, protons belong to the collection of material objects.

Whereas real things exist entirely by themselves, every construct exists in some context or other, e.g. by fiat or by proof in some theory. For example, the natural numbers exist formally in number theory but not in lattice theory. Moreover in standard ("classical") mathematics no attention is paid to the psychology of research and we *pretend* that all the admissible mathematical objects are ready made: mathematics is distinguished from the creation or the learning of it. There is nothing

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to prevent us from inventing such a fiction precisely because mathematical objects are *entia rationis* quite unlike material objects.

How does formal (conceptual) existence relate to real (material, concrete) existence? The relation is one between creators and constructions, i.e. between inquiring persons and their conceptions. All constructs are created by thinking people: no thinking people, no constructs. This is of course a denial of Platonism. (Plato was right in holding that ideas are immaterial, wrong in believing that they exist really by themselves.)

Another possible relation between formal and real existence is the one between a mathematical system and a physical, biological, social or economic one. In other words: how is mathematics related to reality? This is a sub-problem of the general one: what is the relation between ideas and the external world?

If we look for an answer in the history of ideas we may be easily misled, since, by conveniently disregarding counterexamples, we are likely to find cases confirming almost any of our prejudices. Thus from the fact that *some* mathematical ideas have originated in practical concerns, or have ended up being used in science and technology, one might be tempted to "conclude" that *every* mathematical object represents some aspect of reality - the empiricist, pragmatist and vulgar materialistic theses -; or that every thing is identical with, or at least an imperfect copy or realization of, some mathematical object - the objective idealist thesis.

Mathematics is *ontologically noncommittal*, and this is why it can be employed as a tool in constructing theories representing things of many different types - or none. Indeed the same mathematical systems or theories are likely to occur in a great many different research fields, together with a different interpretation. However, such interpretations (or semantic assumptions) are not of pure mathematics: they are part of factual theories (Bunge 1974b, vol. 2, chapter 6, section 3). The propositions in pure mathematics are about purely conceptual objects such as sets and functions.

If mathematics does not represent the world, if it is not the most general science of reality, then in principle it cannot account for change. Mathematical objects are timeless; however, mathematical *activity* is temporal - like any other process. Any mathematical description of real change involves some semantic assumptions whereby certain mathematical objects are assumed to represent nonmathematical objects such as places, time, velocities, or some other properties of real things. Change occurs in the things represented by the mathematical constructs, not in the constructs themselves.

We do not *identify* processes with mathematical operations but assume that the latter can correctly *represent* the former. The representative exists conceptually (formally), the represented materially (actually).

Mathematics and objectivity

Although mathematics is not semantically objective, for it does not represent the external world, it is undoubtedly objective in some other sense, in which art, another fiction, is not. Thus we agree that the rational numbers are denumerable, that the rotations about a fixed point constitute a group, and that some differential equations can be integrated by the method of the Laplace transform. Moreover such agreement is not merely a matter of fashion or arbitrary convention: it is the result of reasoning. Indeed, once certain assumptions have been adopted, we are rationally committed to admitting their logical consequences. By this we do not mean stray assumptions, but hypotheses that form systems (i.e. theories). So, the objectivity of mathematics consists in the lawfulness of its objects, not in that mathematics is a sort of universal physics. Mathematics is *methodologically objective*, not semantically. For example, Zermelo's principle is sometimes stated as "*Every set* can be *well ordered*." However, this formulation is not strictly mathematical: it is a pragmatic interpretation of "There is *a well ordered set containing the elements of any given set*." In this case “there is” designates the concept of conceptual existence. This statement does not refer to subjective experience or action: it only asserts the formal existence of certain mathematical constructs.

Mathematics is *neither* objective *nor* subjective in a semantic sense. From a semantic viewpoint mathematics is neither subjective (intuitionism) nor objective (Platonism) but *neutral*, because it is neither about subjective experience nor about an autonomously existing world. Of course mathematical creation presupposes the existence of creators, i.e. living mathematicians working in a favorable culture. The point is that, although mathematics is created by real beings, it is neither independently real nor, by itself, representative of reality. Yet nothing prevents us from *pretending* that mathematical objects exist in a *sui generis* fashion (i.e. formally), and everything encourages us to use mathematics in our study of reality.

Mathematical objects are thus on a par with artistic or mythological creations: they are all *fictions*. The real number system and the triangle inequality axiom do not exist any more than Don Quixote or Donald Duck. The crucial differences between mathematical fictions and all others are the following:

(i) mathematical objects - such as sets, functions, categories, groups, lattices, Boolean algebras, topological spaces, number systems, differential equations, manifolds and functional spaces - though devoid of factual reference, are not totally free inventions, let alone lies or products of self-deception: they are constrained by laws (axioms, definitions, theorems); consequently they cannot possibly behave “out of character”- e.g. there can be no such thing as a right angle equilateral triangle, whereas even mad Don Quixote is occasionally lucid;

(ii) mathematical objects exist (formally) either by postulation or by proof, never by artistic fiat, and mathematical proofs are purely conceptual procedures;

(iii) mathematical objects are theories or referents of theories, whether full-fledged or in the making, whereas myths, fables, stories, poems, and paintings are non-theoretical;

(iv) mathematical objects and theories are fully rational, not intuitive, let alone irrational (though of course mathematical intuition is acquired by practice);

(v) mathematical statements must be justified in a rational manner, not by intuition or experience;

(vi) far from being dogmas, mathematical theories are based on hypotheses that are given up if shown to lead to contradiction, triviality, or redundancy;

(vii) mathematical theories are linked together forming a super-system; thus logic employs algebraic methods, and number theory resorts to analysis; on the other hand there is no such thing as a coherent system of artistic or mythological creations;

(viii) mathematics is neither subjective nor objective, but ontologically noncommittal; only the process of mathematical invention is subjective, and only living mathematicians are real;

(ix) mathematical objects and theories find application in science, technology, and the humanities;

(x) mathematical objects and theories are socially neutral, whereas myth and art often support or undermine the powers that be.

Mathematics and science and technology

Mathematics is necessary but insufficient to build mathematical models of some cognitive or practical value in science and technology. In addition, some substantive knowledge and some intuition are needed. Otherwise the models will be just mathematical toys. There is a tendency among applied mathematicians to play mathematical games instead of grappling with the complexities of the world. This tendency is obnoxious in the social sciences, and in particular in economics, where many a mathematical model is based on more or less plausible (commonsensical) but entirely arbitrary assumptions that entail “*precisely stated but irrelevant theoretical conclusions*” (Leontief 1982).

In factual science and technology mathematics should be handled as an instrument to build realistic models solving genuine problems. This conception of the nature and role of mathematics is called *instrumentalist formalism*, or *formal instrumentalism* by Bunge. It differs from the instrumentalist (or pragmatist) epistemology in tha

(a) it does state that mathematical formulas are rules or instructions rather than propositions - i.e. category theory is about abstract mathematical systems, set theory is about sets, number theory is about integers, trigonometry is about triangles, topology and geometry are about spaces and so on -;
(b) it does not make practice the value criterion; and consequently
(c) it does not reject the mathematical ideas that have not yet found application, anymore than those that are no longer widely used in science and technology.

A philosophy of mathematics

A philosophy of mathematics should propose well-founded answers to such questions as:

(i) What is mathematics and how does it differ from the other sciences?
(ii) What is the nature of mathematical objects and how do they differ from material object?
(iii) How do mathematical objects exist?
(iv) Does mathematics have any ontological presuppositions?
(v) Is mathematics a priori, a posterior, or both?
(vi) What is mathematical truth?
(vii) What is mathematical proof?
(viii) How does mathematics relate to elementary logic and to semantics?
(ix) How can mathematics, which is not temporal, cope with reality, which is changing?

Bunge's philosophy of mathematics (1985, vol. 7 part I), which he designates as *conceptualist and fictional materialism*

(a) accounts for the purely conceptual nature of mathematical objects;
(b) accounts for the difference between formal and factual propositions, as well as between mathematical proof and empirical validation;
(c) accounts as well for the difference between logical models and the models in science and technology;
(d) accounts for invention of new constructs and discovery of logical relations;
(e) respects the logical stratification of mathematics (elementary logic, category theory[1], set theory, number theory, abstract algebra, topology, analysis, etc).

In Bunge's view (1985), mathematical objects are fictions (classes of brain processes), their mode of production is invention and discovery, their truth is formal, mathematical knowledge is a priori and conceptual. All known mathematical objects are defined (explicitly or implicitly) in purely conceptual ways, without resorting to any factual or empirical means. Mathematical proofs (and refutations) too are strictly conceptual processes making no reference to empirical data. Bunge stresses the anti-Platonist thesis that mathematics does not exist except in the brains of some people.

The Platonic philosophy of mathematics is part of an objective idealist metaphysics, one that postulates the autonomous existence of ideas and their ontological priority. Mathematical *fictionality* is not included in any ontology, because it does not regard mathematical objects as self-existing but as fictions.

How is Bunge's materialistic view compatible with the fictionist component of his philosophy of mathematics, according to which when creating or utilizing a mathematical construct we *pretend* that it leads an impersonal existence? There is no contradiction here, for he holds that it is we, living beings immersed in a concrete society, who construct such fictions. His epistemology is realistic - on scientific realism, see Bunge 1983a,b - concerning the study of the real world and fictionistic concerning fictions. It is also analythically dualistic in that it preserves Leibniz's distinction between *propositions de raison* and *propositions de fait*. Such epistemological dualism does not carry over his monistic ontology because it does not postulate that constructs are part of the furniture of the world. What are found among the furnishings of the world are brains capable of creating constructs, in particular mathematical objects.

So, the mathematical researcher's tasks are to create (or invent) mathematical concepts, propositions, theories, or methods, and to discover their mutual relations, subject only to the conditions of consistency.

Schmitt on the logic of economics

Schmitt (1999) has tackled the problem posed by Husserl in his *The Philosophy of Arithmetic*. According to Husserl, numbers are immaterial objects. According to Schmitt numbers have a real - objective, concrete - existence in economics.

How can numbers exist in economics? Do numbers exist in the real or concrete world? What is the exact definition of numbers which are the transformation of goods? How do goods get transformed into numbers? What is a unit of money? A single sign? These are the questions Schmitt tries to answer.

According to Schmitt, following Husserl's assertion that one unity is not the number one, the unities of goods are transformed into numbers in the process of production.

Furthermore, Schmitt (1986, p. 118) asserts that, "*It was Keynes who discovered that the true unit of economic measurement is not just a number without dimension, but a number which becomes a unit of measurement in the actual operation of wage emission, for it is in the payment of monetary wages that the physical product receives its monetary form.*"

The unit of measurement is the wage unit, because, as Schmitt says, monetary wages define the equivalence of form and substance, i.e. of the product and the number of units of money paid in wages[2].

Real world and numbers in economics

In Schmitt's view, as in Bunge's, numbers have an objective existence. Numbers exist in fact, as prices.

Taking into account the neoclassical thought that goods are integrated into the space of numbers, Schmitt deals with the following problem: how do we link abstract numbers to concrete goods? According to Schmitt, it is true that numbers are immaterial; it is not true to conclude that numbers do not exist in the concrete, since in economics numbers are the form given to goods. In the economic realm goods are changed into numbers, i.e. into wage-units. As Schmitt says goods are integrated in the form of numbers through the mechanism of exchange. This way a commodity is changed or converted into a sum of units of money (Schmitt 1984). Each agent changes its own product into a sum of money. "*To exchange means to* change", says Schmitt (1986, p. 116)

Absolute exchanges integrate goods into the space of numbers

Exchanges are instantaneous operations. Numbers are the form of goods only at the very instant of exchange. Exchanges are flows and stocks are goods (Schmitt 1996).

Following Schmitt we can say that:

(a) no exchange whatsoever is concluded between distinct agents;
(b) therefore, all exchanges are fulfilled between each agent and himself;
(c) each term of an exchange is measured by a number;
(d) in themselves goods are distinct from numbers;
(e) the unit of measurement is the wage unit: monetary wages define the equivalence of form and substance, that is, of the product and the number of units of money paid out in wages.

Instead of providing a linkage between goods and the set of numbers, Schmitt introduces pure numbers into the economy and into macroeconomics, allowing, on a temporary basis, newly produced goods to be replaced by a number of monetary units.

Let us observe the instantaneous replacement of goods by a number of monetary units (Schmitt 1996):

(i) money creation: the banks create + X and - X units of money in one and the same 'impulse';
(ii) the banks pay the factors of production, the agents, by debiting enterprises;
(iii) with respect to the banks, the agents hold a net credit (i.e. a positive sum of money) and the enterprises hold a debit (i.e. a negative sum of money);
(iv) the produced commodity is objectively changed or converted into a sum of X units of money;
(v) monetary income: the agents pay themselves through the enterprises; income defines a physical output in monetary form;
(vi) the agents' monetary income is identical to the agents' physical output which is introduced into a numerical form;
(vii) the enterprises receive the new output, + X , deposited in - X units of money; the enterprises spend a zero-sum of income;
(viii) the enterprises' zero-income expenditure procures a positive income, + X , for the agents.

Thus,

a) the agents change their own product into a sum of money;
b) banks and enterprises are intermediaries;
c) the units of money have no value in themselves.

In fact,

(i) for enterprises, the units of money are pure negative numbers which are made to absorb newly produced commodities; and
(ii) for the agents, the units of money define equivalent positive sums of money which function as forms of physical output.

The *absolute* price of produced commodities is x units of money, because output is contained in x units of money.

The final purchase of the produced commodity is another flow where enterprises are credited and employees debited.

It is obvious that the numerical form of the goods is immaterial and that it is not a part of the stocks. In the absolute exchange defined by production, goods are transformed into numbers. Absolute exchanges are concrete operations; at the very instant that the exchange occurs physical goods become numbers. Thus, in Schmitt's view, numbers are a concrete reality in any absolute exchange. Numbers acquire their economic significance the very instant that producers are paid. Through this exchange, and only then, numbers acquire a concrete existence in the world of economics. In a way, goods "become" concrete numbers.

According to Schmitt, goods, at that very instant of the exchange, effectively assume a numerical form . This truth ought to be treated as an *axiom* in modern economics.

Neoclassical thought and after

According to neoclassical authors, numbers entering in economic exchanges enjoy the same existential status as merchandise. They talk about a kind of "dual space" to accommodate the possible integration of goods into the space of numbers. It is true that without money, economics would not admit the effective presence of pure numbers in exchanges. Since even in economics the numbers are abstract, it is logical to try to introduce goods on a par with numbers, as the neoclassical authors try to do. This attempt is important.

According to neoclassical authors, goods occupy an invariable place in the set of real numbers. Furthermore, the price of the *numéraire* is the number one. In this paradigm, goods are both merchandise and numbers. Hence merchandise exists as such, as goods, and in the space of numbers. This is why they speak about a "dual space", but they are not able to explain how such co-existence comes to be[3].

Neoclassical thought is not able to introduce accounting units in exchanges, as a transformation is needed to change goods into numbers. From the moment goods are introduced into the domain of numbers, they can no longer pertain to the space of merchandise. A neoclassical "dual space" cannot exist simultaneously for numbers and goods.

While neoclassical authors talk about real goods, Schmitt deals with monetary accounting units. According to Schmitt only monetary units are accounting units, i.e. "concrete numbers".

While physical measures are dimensional numbers -i.e. numbers which are related to a physical dimension: mass, velocity, charge or whatever -, economic measures are "a-dimensional", in other words, numbers without any relation to an economic dimension whatsoever. There is no sense to claim that they comprise a preexisting "economic dimension". Rather the relevant point in economics is to express how heterogeneous goods "transubstantiate" into numbers.

As explained by Schmitt (1984), in macroeconomics, the absolute exchange transforms merchandise into actual, objective, concrete numbers. In fact, in order to achieve the transformation of goods into numbers, a "production-exchange" is necessary. That is, only through such very particular kind of exchange - as the one which takes place in production- can merchandise, or products, "become" numbers. To this end, it is necessary to introduce accounting units - "the concrete numbers"- into the flow of production itself.

In the neoclassical paradigm merchandise is a stock, and flows refer to pre-existing goods, stocks which are thus put in motion. But while the neoclassical authors think that transformations of goods into numbers are exchanges whose terms refer to a pre-existing merchandise, Schmitt deals with exchanges in a very different way. According to him, numbers are *concrete entities* immanent to the flow. Wages are paid through an instantaneous flow of money which transforms current output into monetary *income*; furthermore, monetary income comes into being only the instant that payments are made: the first *absolute* exchange. By spending their monetary wages, workers purchase their own real product: the second *absolute* exchange.

Economic exchanges are *instantaneous flows*. Furthermore, the accounting units which are obtained through the sale of one of the terms of the exchange must also be spent instantaneously for the purchase of the other term of that exchange.

In reality, the accounting units are credits-debits towards the banks, i.e. an accounting unit is an *asset-liability*; in this first function of money, bank money are accounting units. Each agent is *simultaneously* credited and debited in monetary accounting units: this is a fact due to the logical strictures of book-keeping.

It is i crucial to distinguish flows from stocks, since only thus can we understand the introduction of pure numbers in economics. The monetary units created by banks as assets-liabilities are coupled to output *via* production through the payment of wages to the factors of production: the only economic flux well defined, since economic production is not a pre-existing product put in motion . Fluxes are prior to stocks and govern them. This is why the accounting units are objective numerical "magnitudes" that inhere in all economies.

According to Husserl the number one is not a unit of whatever (potatoes, pencils, etc.). According to Keynes, the unit of account in economics is the wage unit. To Schmitt, following Keynes, the true monetary unit is the wage unit.

Following Husserl's approach to numbers and unity, the equal entities to be counted are the accounting units. In economics, these units are monetary units, i.e. wage units. In economics, to measure means to (ac)count equal objects. The objects, or outputs, are henceforth measured, i.e. expressed or made into numbers, defined by specified monetary units which are derived from the process of production. Measurement in economics means the counting of monetary units (wage units) .

In short, in macroeconomics in order to measure different goods,

(i) we transform these goods into specifically deployed numbers;
(ii) these numbers are specified by the monetarised accounting units already factored in the economic process of production; and
(iii) these accounting units are wage units.

In a nutshell: in order to appreciate the unique impact of arithmetic on economics, we have to look at the process of production. Goods and numbers are transformed through instantaneous operations, as Schmitt conceptualised already forty years ago .

A Bungean interpretation of Schmitt´s approach to macroeconomics?

We submit that in economics a number is not the sign of a quantity, but a unique construct. Let us say something more about this conception of numbers in economics.

Following Bunge, we distinguish a reality, here economic reality, from factual models about reality; also we distinguish factual models from formal models.

Economists mirror factual exchange by means of a factual model. If macroeconomics were a science like, say, physics, we could say that goods are expressed (and measured) by numbers at the very instant production takes place. This is the important thing in Schmitt's theory: we must say not only that goods "are represented" by numbers, but that goods *become* numbers. In Schmitt's macroeconomic theory goods and money are linked by an *absolute* exchange: goods are transformed, converted into numbers.

In Bunge´s nomenclature, the factual model of production is related to an economic analysis of production. Goods and services take their numerical expression from their link with money: in fact, in the factors market of economic reality, money is linked to goods and services.

In arithmetic, according to Bunge, numbers are fictions. For economics, according to the Schmittian approach to quantum macroeconomics, numbers are real, concrete, as the *absolute* exchange requires. So, in trying to apply the Bungean interpretation of arithmetic to quantum macroeconomics we should have to take into account this important aspect of the absolute exchange. The numerical form of money - numbers - is created by banks *ex-nihilo* using double-entry book-keeping and the purchasing power of a monetary income is created through the process of production. Money has no value of its own as it is created, as an asset-liability, by banks. Income is created by production and defines an absolute exchange between a real and a monetary deposit.

On the one hand, numbers, that is arithmetic, are fictions and have no real existence; on the other, in economics - according to Schmitt's theory - numbers assume a real existence as the numerical form of wages, i.e. numerical expressions of salaries. Following Schmitt we can say not only that goods "are represented" by numbers, but also that goods become numbers. This last statement is what characterizes Schmitt's quantum macroeconomic theory: money and goods are two aspects of the same reality, through the absolute exchange.

So, we could "(re)present" goods in a kind of "dual space": as both physical goods in themselves and as numerical forms (numbers). The exchange between output and monetary wages is an exchange between one object and its monetary form.

Numbers are immaterial, i.e. they are pure forms. Goods are covered by the form of numbers, the form is their "dual" space. Goods become accountable because they get such numerical form. So goods enjoy a double existence, in material space and in the realm of pure numbers.

Goods are introduced, by means of the process of production and through the banking system, in their dual numerical and material space. Goods and their numerical form are a singular thing. According to Schmitt, banks issue money as a numerical form, not as a net asset, and through its close tie to production money acquires a positive value and is thus transformed into *income* (Schmitt 1966,1999).

So, we could say that the reality of the economic exchange *is represented* by a dual space. We have underlined the words "is represented". In fact, while in a strict Bungean interpretation we could say that economic reality (the real exchange) should be distinguished from the factual model of that reality, and in that factual model we should take into account the formality of number theory (a formal model in itself) plus certain ontological assumptions concerning the real economic world - i.e. real goods and real exchange -, Schmitt has shown that economic exchange is of a very particular kind. The numerical expressions of real goods inform their monetary definition: it is their cost of production in wage units. This defines the absolute exchange between money itself and current output. Such absolute exchange occurs between a real good and itself.

By differentiating *money proper* (which is a valueless numerical form) and *monetary income* (which is deposited with banks), money and real goods become two aspects of the same reality. Real goods are transformed into money and money becomes the numerical form - numbers - of real goods.

If we were to read the Schmittian approach to macroeconomics through the Bungean philosophy of mathematics, we could use the neoclassical concept of "dual space" to name - just to label - what Schmitt has so masterfully explained[4]. This has nothing to do with the old fashioned nominalism, since we know perfectly well that concepts are polysemic. Different concepts are to be dealt with within a theoretical framework. Ours is Schmitt's framework.

In principle nothing prevents us from distinguishing economic reality (real exchange) from the factual model about that reality. In doing so, we should note (a) that the factual model "to a certain extent" represents reality, and (b) that this factual model is built upon mathematics (i.e. "numbers") and on some semantic assumptions concerning real economic exchange.

But this Bungean approach can be misleading, since the concept of "dual space" is a very neoclassical one. So, it is much better to read and understand the Schmittian approach in its own terms[5].

The point is that economics, quantum macroeconomics, is a very special kind of science, at the very heart of which lies absolute exchange, which regulates economic exchanges. That is, economics, macroeconomics, is a very particular kind of science and we cannot measure its entities, such as goods, by means of dimensional numbers. We cannot treat it as a "normal" science. Moreover, in Schmitt's words (1996), while mathematics is the formal study of formal entities, "*economics is the formal study of* substantive *entities*." Substantive entities have to be transformed into forms, i.e. into "pure" numbers.

Logic and economics

As mentioned above, mathematical logic is the formal study of formal entities and belongs to what is known as symbolic logic. In fact, mathematics is a formal science, for it is the theoretical study of numbers, which are "forms" by definition.

Economics is the formal study of substantive entities, such as goods. The forms which contain these substantive economic entities have to be studied in a special manner, through production-exchange, i.e. through an *absolute* exchange (Schmitt 1984). This unique use of numbers marks the entire economic process: goods and numbers become related through production.

In economics we study numbers, i.e. forms, meant to contain - in the Schmittian sense of the term - real substantive entities. The different heterogeneous goods have first to be transformed into numbers, through their association with money. Then we may claim that these numbers can be studied by a special kind of logic, a conceptual logic that, following Schmitt, could be called *economic logic*. Absolute exchanges are not derived from mathematical analysis. They belong to an economic analysis of the production process.

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[1] The foundational landscape of mathematics was changed in the 1960s. Lawvere (1966) discovered that the concepts of set and membership which are basic (undefined, primitive) in set theory, are definable in another theory, namely category theory, founded in 1945 (MacLane 1971).The basic notions of this theory are those of morphism and morphism composition (Lawvere and Schanuel 2000). Sets have turned out to constitute just a model of a category; i.e. categories are more abstract, fundamental and inclusive mathematical objects than sets.

[2] In Keynes' words (1973, p. 41), "*We shall call the unit in which the quantity of employment is measured the labour-unit; and the money-wage of labour-unit we shall call the wage-unit*."

[3] According to Schmitt (1999, p. 25), "*L'erreur fondamentale commise par les auteurs néoclassiques est facile à caractériser: ils pensent (...) que les opérations cruciales, transformation de bien réels in nombres, sont des échanges dont les termes sont des marchandises préexistantes*."

[4] Schmitt (1999, p. 24) has explained this transformation through the production-exchange: "*Une bonne façon de montrer l'erreur de la pensée néoclassique est justement d'arguer qu'aucune opération, concrète ni même abstraite, ne peut réussir la transformation de marchandises préexistantes en nombres (...) un échange tout à fait particulier, "échange de production", est nécessaire pour que les marchandises, plus précisément les produits, deviennent des nombres*."

[5] If we wanted to maintain the concept of "dual space" we should say, with Schmitt (1999, p. 22) that,*" Le logicien de l'économie fait (...) avancer la logique de la philosophie; il est vrai que les nombres sont incorporels, immatériels; il est faux d'en déduire que les nombres n'existent pas dans le concret; (...) en économie les nombres sont les formes (numériques) des marchandises. Au moment précis où les marchandises sont introduites dans leur espace dual, numérique, elles sont simultanément des objets matériels, biens et services, et des nombres ; il serait risible de prétendre séparer un objet de la forme qu'il revêt ; une marchandise et sa forme numérique c'est tout un.*